NONLINEAR LONGITUDINAL WAVES IN A ROD TAKING ACCOUNT OF THE INTERACTION OF STRAIN AND TEMPERATURE FIELDS

A. I. Potapov and N. P. Semerikova

UDC 539.3+536.2

In a number of problems on the propagation of longitudinal thermoelastic waves in a rod, it is very important to take account of convective heat exchange with the surrounding medium, which in some cases may even predominate over thermal conductivity effects. For example, similar effects are encountered in measuring the frequency relationship for a material in rod specimens [1, 2], and also with high-frequency methods for predicting the supporting capacity of structural materials on a large base of cyclic loading [3].

Here we study the effect of thermal conductivity and heat exchange through the side surface of a rod on the evolution of nonlinear longitudinal waves. It has been demonstrated that in metals for wavelengths large compared with the rod diameter, thermal conductivity effects are negligibly small compared with conductive heat exchange. In this case the effect of heat exchange appears to be similar to oscillation energy dissipation in a rod made of viscoelastic material followed by relaxation [4]. Solutions have been studied describing nonlinear steady-state and quasi-steady-state waves running in a single direction. Equations have been obtained for measuring the energy and amplitude of running waves and their partial solutions have been analyzed.

Propagation of a nonlinear longitudinal wave in an infinite cylindrical rod, which in the original condition is in thermal equilibrium with the surroundings, is considered. With passage of a longitudinal wave, temperature drops occur between regions of compression and tension in the material, which leads to development of heat flow both within the rod and through its side surface [5, 6]. It is assumed that the length of the propagating waves is greater than the rod diameter, temperature is the same over its cross section, and thermoelastic effects are small and are only retained in a linear approximation. Movement of rod particles in the radial direction is also considered. With these assumptions the nonlinear dynamic thermoelasticity problem is described by the following set of equations:

$$u_{,tt}-\left(1+\frac{6\alpha\varepsilon_0}{E}u_{,x}\right)u_{,xx}-\frac{\nu^2a^2}{2\Lambda^2}\frac{\partial^2}{\partial x^2}\left(u_{,tt}-\frac{c_\tau^2}{c_s^2}u_{,xx}\right)=-\frac{\alpha_T\Theta}{\varepsilon_0}\frac{\partial}{\partial x}(T-T_0); \qquad (1)$$

$$T_{,t}-\frac{\chi}{c_s\Lambda}T_{,xx}+\frac{2h\Lambda}{a\rho_0c_pc_s}(T-T_0)=-\frac{\alpha_TT_0c_s^2\varepsilon_0}{c_p\Theta}u_{,xt}. \qquad (2)$$

Here $x = x'/\Lambda$, $t = c_S t'/\Lambda$, $u = u'/\Lambda\varepsilon_0$, $T = T'/\Theta$ are dimensionless variables; x' is dimensional spatial coordinate; t' is time; a is rod radius; Λ is wavelength; $\alpha=E^{-1}\left[\frac{(1-2\nu)^3}{6}\nu_1+(1-2\nu)\nu_2+\frac{4}{3}\nu_3\right]$ is a dimensionless coefficient of nonlineariy; $\nu_{1,2,3}$ are third-order elasticity moduli; E and ν are Young's modulus and Poisson's ratio; ρ_0 is material density; ε_0 is characteristic strain; u'(x, t) is axial displacement; $c_S = (E/\rho_0)^{1/2}$, $c_\tau = (\mu/\rho)^{1/2}$ are longitudinal and shear wave velocities in the rod; T is current rod temperature; T_0 is temperature of the surroundings; θ is characteristic scale of temperature variation; χ, c_p, h, α_T are thermal diffusivity, specific heat capacity, heat exchange, and thermal expansion coefficients, respectively.

For a rod of steel-grade Hecla 138 A with parameters [7, 8] $\varkappa = 45.4$ W/(m·°K), $c_p = 460$ J/(kg·°K), $h = 10^2$ W/(m²·°K), $E = 2\cdot10^{11}$ Pa, $\rho_0 = 7.8\cdot10^3$ kg/m³, $\nu = 0.29$, $\nu_1 = -3.23\cdot10^{11}$ Pa, $\nu_2 = -2.65\cdot10^{11}$ Pa, $\nu_3 = -1.77\cdot10^{11}$ Pa, $\alpha_T = 10^{-5}$ K^{-1} with strains $\varepsilon_0 \sim 10^{-5}$, dimensionless coefficients in (1) and (2) are ($T_0 = 300$ K, $\Theta \approx \alpha_T^{-1}\varepsilon_0 \sim 10^{-5}$ K, $2a = 1$ cm) $6\alpha\varepsilon_0/E \approx -5\cdot10^{-6}$, $\nu^2a^2/(2\Lambda^2) \approx 5\cdot10^{-2}\ (a/\Lambda)^2$, $\alpha_T\Theta/\varepsilon_0 \sim 10^{-5}$, $\chi/(c_S\Lambda) \approx 2.6\cdot10^{-9}\ (\Lambda\ \text{cm})^{-1}$, $2h\Lambda/(a\rho_0c_pc_S) \sim 10^{-9}\ (\Lambda/a)$, $\alpha_Tc_S^2T_0\varepsilon_0/(c_p\Theta) \sim 160$. From the estimates provided it follows that in a rod of diameter $2a = 1$ cm with $\Lambda \geq 20\,a$, heat exchange through the side surface predominates over thermal con-

Gorkii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 57-61, January-February, 1988. Original article submitted October 3, 1986.

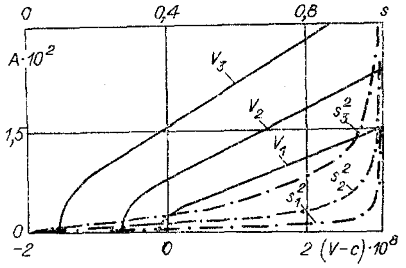

Fig. 1

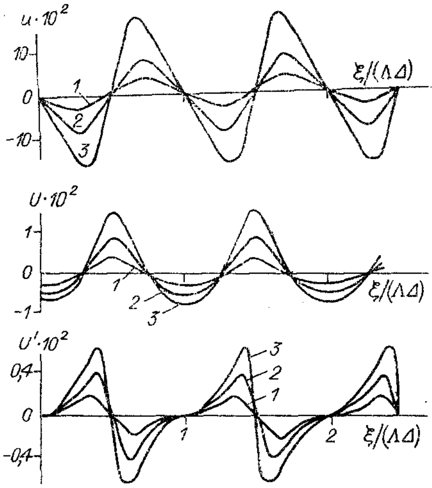

Fig. 2

ductivity. In fact, this situation should occur in the rod coverters of powerful ultrasonic systems [9, 10]. The reverse situation will occur with excitation of short pulses (solitons) in a wire of diameter $2a = 1$ mm with $\Lambda < 10\,a$ [11]. In the case when heat exchange markedly predominates over thermal conductivity, set (1), (2) is reduced to a single equation

$$u_{,tt} - (1 + 6\alpha\varepsilon_0/E\cdot u_{,x})\,u_{,xx} - \nu^2 a^2/(2\Lambda^2)\frac{\partial^2}{\partial x^2}(u_{,tt} - v^2 u_{,xx}) = \tag{3}$$

$$= h_*^{-1}\frac{\partial}{\partial t}\Big\{u_{,tt} - (c^2 + 6\alpha\varepsilon_0/E\cdot u_{,x})\,u_{,xx} - \nu^2 a^2/(2\Lambda^2)\frac{\partial^2}{\partial x^2}(u_{,tt} - v^2 u_{,xx})\Big\},$$

$$c^2 = 1 - 2\alpha_T^2 T_0 c_s^2/c_p, \quad v = c_\tau/c_s, \quad h_* = 2h\Lambda/(a\rho_0 c_p c_s),$$

where a parameter determining energy dissipation in the rod is the dimensionless heat exchange coefficient h_*. In the limiting case ($h_* = 0$, i.e., a thermally insulated rod), from (3) a nonlinear wave equation is obtained for adiabatic processes, and with $h_* = \infty$ for isothermal processes. In real situations $h_* \sim 10^{-8}$, and therefore the presence of heat exchange may be considered as a small deviation of wave processes from adiabatic processes. Considering that in the zero approximation $u_{,tt} \approx c^2 u_{,xx}$, we lead (3) to the form

$$u_{,tt} - \left(c^2 + \frac{6\alpha\varepsilon_0}{E}u_{,x}\right)u_{,xx} - \frac{\nu^2 a^2}{2\Lambda^2}(c^2 - v^2)\,u_{,xxxx} + h_*(1 - c^{-2})\,u_{,t} - \frac{h_*\nu^2 a^2(c^2 - v^2)}{2\Lambda^2 c^2}u_{,xxt} = 0. \tag{4}$$

Here the term proportional to $u_{,t}$ and $u_{,xxt}$ determines the low-frequency and high-frequency losses, respectively. In [2] it was shown that heat exchange through the side surface of a rod leads to a rheological equation for the material relating to the model of a standard viscoelastic body [1].

For long-wave perturbations ($\Lambda/a > 1$), it is possible to introduce new (wave) variables $U_{1,2}$ [9]

$$u_{,x} = U_1 + U_2, \quad u_{,t} = c(U_1 - U_2) + \frac{\nu^2 a^2}{2\Lambda^2 c}\frac{\partial^2}{\partial x^2}(U_1 - U_2), \tag{5}$$

reducing Eq. (4) to a symmetrical set of equations for connected normal waves

$$\frac{\partial U_{1,2}}{\partial t} \mp c\frac{\partial U_{1,2}}{\partial x} \mp \beta\frac{\partial^3 U_{1,2}}{\partial x^3} \mp \alpha_{\text{н}}\frac{\partial}{\partial x}(U_1 + U_2)^2 = \pm\zeta(U_1 - U_2) + \delta\frac{\partial^2}{\partial x^2}(U_1 - U_2), \tag{6}$$

where the fucntions U_1 and U_2 describe strain waves running counter to each other; $\alpha_H = 3\alpha\varepsilon_0/E$, $\beta = \nu^2 a^2(c^2 - v^2)/4\Lambda^2$ are coefficients of nonlinearity and dispersion; $\zeta = h_*(1 - c^{-2})$, $\delta = h_*\nu^2 a^2(c^2 - v^2)/2\Lambda^2 c$ are coefficients for low-frequency and high-frequency losses. In a linear conservative approximation, U_1 and U_2 are independent and only interact as a result of nonlinearity and dissipation.

We consider evolution of a wave $U_2 = U(x, t)$, running in the direction opposite of the x axis (single-wave approximation $U_1(x, t) = 0$). Since thermoelastic effects are small, then as an unperturbed solution it is possible to choose a nonlinear wave propagating in an elastic material. In this case $\zeta = \delta = 0$, and system (6) is reduced to a Kortewege—de Vries equation, which permits solution in the form of running steady-state waves. In the general case the form of the latter is nonsinusoidal, and it depends on the ratio of values for the nonlinearity α_H and material dispersion β. Steady-state periodic waves and the strain $U(\xi)$, depending on a single running variable $\xi = x - Vt$, are described by an expression [9]

$$U(\xi) = -\frac{2A(\mathrm{K}-\mathrm{E})}{s^2\mathrm{K}} + 2A\,\mathrm{sn}^2\left[\left(-\frac{\alpha_H A}{3\beta s^2}\right)^{1/2}\xi,\ s\right]. \tag{7}$$

Here A is amplitude; V is wave velocity; s is coefficient of nonlinear distortions; K(s), (s) are first- and second-order whole elliptical integrals. Parameters V, Λ and A for this wave are connected by the relationships

$$V = c - \frac{48\beta \mathrm{K}^2}{\Lambda^2}\left(1 - \frac{\alpha_H \Lambda^2 A}{3} - \frac{\mathrm{K}-\mathrm{E}}{\mathrm{K}}\right); \tag{8}$$

$$\Lambda = \sqrt{12s\mathrm{K}}/\sqrt{-\alpha_H A/\beta}. \tag{9}$$

Equations (7)-(9) make it possible to construct a relationship between A, V and s with different Λ and to determine the shape of the wave for strains $U(\xi)$. Given in Fig. 1 is the dependence of the nonlinear distortion coefficient and wave velocity on amplitude for a rod made of steel Hecla 138 A with Λ = 60 *a*, 45 *a*, 37 *a*. Shown in Fig. 2 is the distribution of displacements $u(\xi)$, strains $U(\xi)$, and strain rates $U'(\xi)$ along coordinate ξ with Λ = 45 *a* and $A = 0.4\cdot10^{-2}$, $0.7\cdot10^{-2}$, $1\cdot10^{-2}$ (lines 1-3). With small amplitudes the shape of all of the waves is sinusoidal. With $A > 10^{-2}$ the shape of the displacement wave is close to saw-tooth, and the strain wave U is a sequence of pulses. Expression (7) describes two different classes of wave: nonlinear quasi-harmonic waves with small amplitudes and essentially nonsinusoidal waves (quasisolitons) with large amplitudes [9-11].

By drawing attention to the smallness of dissipation effects, we limit consideration to nonlinear quasi-steady-state waves described by expression (7) with slowly changing parameters A, V, Λ. It is assumed that as before they are connected by relationships (8) and (9). We use an equation for the change in energy [11], which for the case in question has the form

$$\frac{dw}{dt} = -2\zeta w - 2\delta\int_0^{\Lambda} U_{,x}^2\,dx. \tag{10}$$

Here $w = \int_0^{\Lambda} U^2 dx$ is the total wave energy. It is noted that in the right-hand part of (10) there are only dissipative terms characterizing low-frequency and high-frequency losses. By expressing energy w and the integral in the right-hand part of (10) in terms of wave amplitude A, we arrive at an equation

$$\left(2d_1 A^{1/2} + \frac{3}{2}d_2\right)\frac{dA}{dt} = -2\zeta\left(d_1 A^{3/2} + d_2 A\right) - 2\delta d_3 A^2, \tag{11}$$

where

$$d_1 = \frac{4(\mathrm{K}-\mathrm{E})^2\Lambda}{s^4\mathrm{K}^2};\quad w = d_1 A^2 + d_2 A^{3/2};$$

$$d_2 = 4\left(\frac{3\beta s^2}{-\alpha_H}\right)^{1/2}\int_0^{\Lambda/\Delta}\left[\mathrm{sn}^4(\xi, s) - \frac{2(\mathrm{K}-\mathrm{E})}{s^2\mathrm{K}}\mathrm{sn}^2(\xi, s)\right]d\xi;$$

$$d_3 = 16\left(-\frac{\alpha_H}{3\beta s^2}\right)^{1/2}\int_0^{\Lambda/\Delta}\mathrm{sn}^2(\xi, s)\,\mathrm{cn}^2(\xi, s)\,d\xi;\quad \Delta = \left(\frac{3\beta s^2}{-\alpha_H A}\right)^{1/2}.$$

In the linear case $A \to 0$, analysis of solutions for Eq. (11) leads to the well-known result of exponential wave amplitude attenuation $A = A_0 e^{-gt}$ with $g = \zeta + 4\,\delta\omega^2(1 - 4\pi^2\Lambda^2\beta)$, when the wave is close to a soliton, $w = \frac{4}{3}\left(-\frac{3\beta}{\alpha_H}\right)^{1/2} A^{3/2}$ and (11) takes the form

$$\frac{dA}{dt} = -\frac{4}{3}\zeta A + \frac{14}{3}\frac{\alpha_H \delta}{\beta}A^2. \tag{12}$$

The change in soliton amplitude occurs by a more complex rule [12]

$$A(t) = \frac{A_0 e^{-\frac{4}{3}\zeta t}}{1 - \frac{8\delta\alpha_H}{3\beta\zeta}A_0\left(1 - e^{-\frac{4}{3}\zeta t}\right)} \tag{13}$$

(A_0 is initial amplitude). Whence it follows that low-frequency and high-frequency losses lead to different rules for the change in wave amplitude. For the example given, an estimate of coefficients in Eq. (12) indicates that $\zeta \sim 10^{-12}(\Lambda/a)$ and $14\ \alpha_H \delta/3\beta \sim 2\cdot 10^{-14}(\Lambda/a)$. Consequently, with strains $A \leqslant 1(u,_x \leqslant 10^{-5})$ high-frequency losses are negligibly small, and therefore it is possible to ignore the second term in the denominator of (13), and as a result of this the soliton amplitude will also decrease by an exponential rule but with another decrement.

The authors thank A. I. Vesnitskii for considering the work.

LITERATURE CITED

1. V. S. Postnikov, Internal Friction in Metals [in Russian], Metallurgiya, Moscow (1974).
2. Ya. S. Podstrigach and R. N. Shvets, "Theory of thermoelastic friction in solids," in: Internal Friction in Metals and Alloys [in Russian], Nauka, Moscow (1966).
3. V. A. Kuz'menko, Sonic and Ultrasonic Vibrations with Dynamic Testing of Materials [in Russian], Izd. Akad. Nauk UkrSSR, Kiev (1963).
4. Yu. N. Rabotnov, Elements of Inherent Mechanics of Solids [in Russian], Nauka, Moscow (1977).
5. U. K. Nigul and Yu. K. Éngel'brekht, Nonlinear and Linear Transition Wave Processes of Deformation for Thermoelastic Bodies [Russian translation], Izd. Akad. Nauk ÉstSSR, Tallin (1972).
6. Ya. S. Podstrigach and R. N. Shvets, "Dynamic problem of thermoelasticity for a thin rod taking account of heat exchange from its surface," in: Mechanics Questions for Real Solid [in Russian], 2nd ed. Naukova Dumka, Kiev (1964).
7. G. I. Pogodin-Alekseev (ed.) Handbook of Engineering Materials, in 4 vols. [in Russian], Mashgiz, Moscow (1959).
8. A. I. Lur'e, Nonlinear Elasticity Theory [in Russian], Nauka, Moscow (1980).
9. I. V. Miloserdova and A. I. Potapov, "Nonlinear standing waves in a rod of finite length," Alust. Zh., 29, No. 4. [in Russian], Mashgiz, Moscow (1983).
10. I. V. Miloserdova and A. I. Potapov, Vibration in nonlinear rods with presence of internal resonances," Mashinovedenie, No. 4 (1983).
11. L. A. Ostrovskii and A. M. Sutin, "Nonlinear elastic waves in a rod," Prikl. Mat. Mekh., 41, No. 3 (1977).
12. L. A. Ostrovsky, "Solitons in active media," in: Nonlinear Deformation Waves, U. Niguel and J. Engelbrecht (eds.), Springer, Berlin (1983).

ANALYSIS OF THE POWDER COMPACTION PROCESS IN A CYLINDRICAL CONTAINER ON THE BASIS OF A SIMPLE MODEL

A. I. Matytsin

UDC 539.374

Explosive compaction of powders is often accomplished in cylindrical geometry when the applied load is quite large and, as a result of this loading, may affect the strength properties of materials. A similar point of view was expressed in [1], and this is also indicated by experimental results [2-4]. During shock loading the final powder density, shock wave (SW) amplitude, and the strength properties of the compacted material appear to be connected with each other in a complex fashion. However, since the main change in powder volume occurs in the shock-wave front (SWF), as a first approximation the change in density behind the front is ignored, and it is assumed to be constant. In addition, there is one more severe simplification, i.e., the dynamic yield strength is assumed to be constant. With detonation rates much greater than the SW velocity in the powder, the slope of it to the container axis is small, and for analysis it is possible to use a unidimensional model.

In a unidimensional arrangement the problem of loading a compacting cylinder without a shell was resolved in [5]; the case was studied numerically for constant load and dynamic yield strength depending linearly on internal energy of the material, and also the asymptotic behavior was found for SW amplitude at the start and end of the process of its convergence.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 61-70, January-February, 1988. Original article submitted October 22, 1986.